

Creation of unstable particles and decoherence in semiclassical cosmology

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Abstract

We consider a simple cosmological model in order to show the importance of unstable particle creation for the validity of the semiclassical approximation. Using the mathematical structure of rigged Hilbert spaces we show that particle creation is the seed of decoherence which enables the quantum to classical transition.

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I. INTRODUCTION

We know that the laws of classical mechanics describe with a high degree of accuracy the behavior of macroscopic systems. And yet, it is believed that phenomena on all scales, including the entire Universe, follow the laws of quantum mechanics. So, if we want to reconcile our two last statements, it is essential to understand the transition from the quantum to the classical regime. One of the scenarios where this problem is relevant is quantum cosmology, in which one attempts to apply quantum mechanics to cosmology. This involves a problem that has not been solved; namely, quantizing the gravitational field. Therefore as a first attempt, it is an important issue to predict the conditions under which the gravitational field may be regarded as classical.

The quantum to classical transition is a very old and interesting problem relevant in many branches of physics. It involves the concepts of *correlations*, i.e., the Wigner function of the quantum system should have a peak at the classical trajectories [1], and *decoherence*, that is, there should be no interference between classical trajectories [2]. The density matrix should be approximately diagonal. In order to understand the emergence of classical behaviour, it is essential to consider the interaction between system and environment, since both the decoherence process and the onset of classical correlations depend strongly on this interaction. Both ingredients are not independent and excess of decoherence can destroy the correlations [3].

In a previous work [4], one of us has studied the problem of choosing an alternative mathematical structure, based on a new spectral decomposition with generalized unstable states, which is useful to explain time asymmetry of different models. Following [4], we will show that this unstable quantum states satisfy correlation conditions and also produces decoherence between different cosmological branches. From this work, we know that if we want to retain the time-symmetric laws of nature and at the same time explain the time asymmetry of the universe, we must choose a space of solutions which is not time-symmetric. A convenient choice of time-asymmetric spaces was already proposed in Ref. [5].

The scheme is based in the existence of a physically admissible quantum superspace Φ_- and therefore also the existence of a superspace of time inverted states of Φ_- , namely a physically forbidden quantum superspace Φ_+ . Thus, the time inversion that goes from Φ_- to Φ_+ is also forbidden [4]. If the generalized states in Φ_- are restricted to be included in the superspace of regular states \mathcal{S} (and the same for Φ_+ with \mathcal{S}^\times where \mathcal{S}^\times is the space of an (anti)linear functional over \mathcal{S}), our real mathematical structure is the Gel'fand triplet (or rigged hilbert space) [4]:

$$\mathcal{S} \subset \mathcal{H} \subset \mathcal{S}^\times. \quad (1)$$

If K is the Wigner time-reversal operator we have

$$K : \Phi_- \rightarrow \Phi_+ \quad ; \quad K : \Phi_+ \rightarrow \Phi_-. \quad (2)$$

Using these spaces of “generalized” states we can also find time-asymmetry for the generalized states. If we choose Φ_- as in Ref. [4], Eq. (2) means that these generalized states will be (growing or decaying) Gamow vectors. Decaying states are transformed into growing states (or vice-versa) by time-inversion.

As we have said [4], the choice of Φ_- (or Φ_+) as our space of quantum states implies that K is not defined inside Φ_- (or Φ_+), so that time-asymmetry naturally appears.

But, in the cosmological case, the choice between Φ_- or Φ_+ (or between the periods $t > 0$ or $t < 0$, or between the two semigroups) is conventional and irrelevant, since these objects are identical (namely one can be obtained from the other by a mathematical transformation), and therefore the universes, that we will obtain with one choice or the other, are also identical and not distinguishable. Only the names *past* and *future* or *decaying* and *growing* will change but physics is the same, i.e., we will always have equilibrium, decoherence, growing of entropy, etc. toward, what we would call the future. But once the choice is made, a substantial difference is established in the model: using Φ_- it can be proved that the time evolution operator is just $U(t) = e^{-iHt}$, $t > 0$, and cannot be inverted (if the choice would be Φ_+ the condition would change to $t < 0$). Therefore even if we continue using the same

reversible evolution equations, the choice of Φ_- (or which is the same Φ_+) introduces time-asymmetry, since now we are working in a space where future is substantially different than past. Thus the arrow of time is not put *by hand* since the choice between the period $t > 0$ and $t < 0$ or between Φ_- and Φ_+ is trivial and unimportant (namely to chose the period $t > 0$ as the physical period and consider $t < 0$ as non-existent, because the period before the “creation of the Universe” is physically unaccessible to us or viceversa). The important choice is between \mathcal{H} (the usual Hilbert space) and Φ_- (or Φ_+) as the space of our physical states. And we are free to make this choice, since a good physical theory begins by the choice of the best mathematical structure that mimic nature in the most accurate way.

As far as we know the new formalism is mathematically rigorous and the physical results of both ones are the same. Two of us have shown this method applied to a semiclassical Robertson-Walker metric coupled to a quantum field [6]. In this article we have shown how to implement this formalism in a semiclassical cosmological model in order to prove the validity of the semiclassical approximation. Decoherence and correlations are two necessary ingredients to obtain classical behaviour. In Ref. [6] we have proved that the model satisfies both requirements for classicality. However, paper [6] was the first step to prove our mathematical structure in a simple cosmological model; we can rise two relevant observations about the validity of the semiclassical approximation:

- 1) considering the infinite set of unstable modes leads to perfect decoherence, destroying correlations [2,10], as we will prove here.
- 2) the existence of correlations was proved for only one mode of the scalar field and not for the entire density matrix.

In the present article we complete and improve our previous work in order to obtain the semiclassical limit as a consequence of the real “balance” between decoherence and correlations.

In the context of semiclassical cosmology from a fully quantized cosmological model, the cosmological scale factor can be defined as $a = a(\eta)$, with η the conformal time. When

$\eta \rightarrow \infty$ we will obtain a classical geometry $g_{\mu\nu}^{out}$ for the Universe. In the semiclassical point of view, the Wheeler-De-Witt equation splits in a classical equation for the spacetime metric and in a Schrödinger equation for the scalar field modes, with the corresponding hamiltonian $h(a_{out})$. Using $h(a_{out})$ and the classical geometry $g_{\mu\nu}^{out}$ we can find a semiclassical vacuum state $|0, out\rangle$ which diagonalizes the hamiltonian; and the creation and annihilation operators related to this vacuum and the corresponding Fock spaces.

In this paper, we choose time-asymmetric Fock spaces to study a simple cosmological model; we analyze how this model fulfills the two requirements for classicality.

The organization of this paper in the following. In section II we introduce the cosmological model and we summarize our previous results of Refs. [4] and [6]. In section III we analyze the conditions for the existence of decoherence and correlations in this model. Since we achieve perfect decoherence, in Section IV we need to introduce a cutoff. We suggest a particular value for the cutoff using a relevant physical scale that ensures the validity of the semiclassical approximation, namely the Planck scale. In section V we briefly discuss our results.

II. THE MODEL AND PREVIOUS RESULTS

In this Section we will only extract the main results of Ref. [6]. Let us consider a flat Robertson-Walker spacetime coupled to a massive conformally coupled scalar field. In the specific model of [6] we have considered a gravitational action given by

$$S_g = M^2 \int d\eta \left[-\frac{1}{2} \dot{a}^2 - V(a) \right], \quad (3)$$

where M is Planck's mass, $\dot{a} = \frac{da}{d\eta}$ and $V(a)$ is the potential function that arises from a spatial curvature, a possible cosmological constant and, eventually a classical matter field.

In this paper we will consider the potential function used by Birrell and Davies [7] to illustrate the use of the adiabatic approximation in an asymptotically non-static four dimensional cosmological model:

$$V(a) = \frac{B^2}{2} \left(1 - \frac{A^2}{a^2} \right), \quad (4)$$

where A and B are arbitrary constants.

The Wheeler-DeWitt equation for this model is:

$$H\Psi(a, \varphi) = (h_g + h_f + h_i) \Psi(a, \varphi) = 0, \quad (5)$$

where

$$h_g = \frac{1}{2M} \partial_a^2 + M^2 V(a), \quad (6)$$

$$h_f = -\frac{1}{2} \int_k \left(\partial_{\varphi_k}^2 - k^2 \varphi_k^2 \right) dk, \quad (7)$$

$$h_i = \frac{m^2 a^2}{2} \int_k \varphi_k^2 dk, \quad (8)$$

and m is the mass of the scalar field.

In the semiclassical approximation, where the geometry is considered as classical, and only the scalar field is quantized, we propose a WKB solution to the Wheeler-DeWitt equation:

$$\Psi(a, \varphi) = \chi(a, \varphi) \exp \left[i M^2 S(a) \right], \quad (9)$$

where S is the classical action for the geometry.

To leading order (i.e. M^2), we get:

$$\left[\frac{dS(a)}{da} \right]^2 = 2V(a), \quad (10)$$

which is essentially the Hamilton-Jacobi equation for the variable $a(\eta)$. From this equation we can find the classical solutions

$$a(\eta) = \pm \left(A^2 + B^2 \eta^2 \right)^{\frac{1}{2}} + C, \quad (11)$$

where C is a constant.

Taking the following order in the WDW equation, we obtain a Schrödinger equation for $\chi(a, \varphi)$:

$$i \frac{d}{d\eta} \chi(a, \varphi) = -\frac{1}{2} \int_k \left[\partial_k^2 - \Omega_k^2 \varphi_k^2 \right] dk \chi(a, \varphi), \quad (12)$$

where $\Omega_k^2 = m^2 a^2 + k^2$

Since the coupling is conformal we will have well-defined vacua [7]. So, we consider now two scales a_{in} and a_{out} such that $0 < a_{in} \ll a_{out}$. Next, we define the corresponding $|0, in\rangle, |0, out\rangle$ vacua there, where $|0, in\rangle$ is the adiabatic vacuum for $\eta \rightarrow -\infty$ and $|0, out\rangle$ is the corresponding for $\eta \rightarrow +\infty$. It is well known [7,8] that, in the case we are considering, we can diagonalize the time-dependent Hamiltonian (Eq. (12)) at a_{in} and a_{out} , define the corresponding creation and annihilation operators, and the corresponding Fock spaces.

Thus, following Eqs. [37 – 43] from Ref. [4] we can construct the Fock space and find the eigenvector of $h(a_{out})$, as follows:

$$h(a_{out}) |\{k\}, out\rangle = h(a_{out}) |\varpi, [k], out\rangle = \Omega(a_{out}) |\{k\}, out\rangle = \sum_{k \in \{k\}} \Omega_{\varpi}(a_{out}) |\varpi, [k], out\rangle, \quad (13)$$

where $[k]$ is the remaining set of labels necessary to define the vector unambiguously and $|\varpi, [k], out\rangle$ is an ortonormal basis [4].

In the same way we can find the eigenvectors of $h(a_{in})$. Thus we can also define the S matrix between the in and out states (Eq. 44 of Ref. [4]):

$$S_{\varpi, [k]; \varpi', [k']} = \langle \varpi, [k], in | \varpi', [k'], out \rangle = S_{\varpi, [k]; [k']} \delta(\varpi - \varpi') \quad (14)$$

As we have explained in the Introduction, we will choose time-asymmetric spaces in order to get a better description of time asymmetry of the universe. Therefore we make the following choice: for the in Fock space we will use functions $|\psi\rangle \in \Phi_{+, in}$ namely, such that $\langle \varpi, in | \psi \rangle \in S|_{R_+}$ and $\langle \varpi, in | \psi \rangle \in H_+^2|_{R_+}$ where H_+^2 is the space of Hardy class functions from above; and for the out Fock space we will use functions $|\varphi\rangle \in \Phi_{-, out}$ such that $\langle \varpi, out | \varphi \rangle \in S|_{R_+}$ and $\langle \varpi, out | \varphi \rangle \in H_-^2|_{R_+}$. So we can obtain a spectral decomposition for the $h(a_{out})$ (in a weak sense) [4,6]:

$$h(a_{out}) = \sum_n \Omega_n |\bar{n}\rangle \langle \bar{n}| + \int dz \Omega_z |\bar{z}\rangle \langle \bar{z}|, \quad (15)$$

where $\Omega_n = m^2 a^2 + z_n$ and z_n are the poles of the S matrix.

From references [4] and [6] it can be seen that S matrix corresponding to this model has infinite poles and the mode k , corresponding to each pole reads:

$$k^2 = mB \left[-\frac{mA^2}{B} - 2i \left(n + \frac{1}{2} \right) \right]. \quad (16)$$

Thus we can compute the squared energy of each pole:

$$\Omega_n^2 = m^2 a^2 + mB \left[-\frac{mA^2}{B} - 2i \left(n + \frac{1}{2} \right) \right]. \quad (17)$$

The mean life of each pole is:

$$\tau_n = \frac{\sqrt{2}}{2} \frac{\left[m^2 (a_{out}^2 - A^2) + \left(m^4 (a_{out}^2 - A^2)^2 + 4m^2 B^2 \left(n + \frac{1}{2} \right)^2 \right)^{\frac{1}{2}} \right]^{\frac{1}{2}}}{\text{Im } B \left(n + \frac{1}{2} \right)}. \quad (18)$$

Using the spectral decomposition (15) we will show, in the next section, how decoherence produces the elimination of all quantum interference effects. But we must notice that we can introduce this spectral decomposition only using the unstable ideal states.

We believe that our results can be generalized to other models, since essentially they are based in the existence of an infinite set of poles in the scattering matrix. Nevertheless the model considered in this paper will allow us to complete all the calculations, being therefore a good example of what can be done with our method.

III. PERFECT DECOHERENCE AND NO CORRELATIONS

In this section we will show how the complete set of unstable modes destroy quantum interference, but also demolish classical correlations. The appearance of decoherence coming from the spectral decomposition of Eq. (15) shows the importance of the unstable modes in the quantum to classical process. It has been proved [9] that decoherence is closely related to another dissipative process, namely, particle creation from the gravitational field

during universe expansion. In Eq. (15) we obtain as in [6] a set of discrete unstable states, namely, the unstable particles, and a set of continuous stable states (see Eq. (15)), the latter corresponding to the stable particles.

As the modes do not interact between themselves we can write:

$$\chi(a, \varphi) = \prod_{n=1}^{\infty} \chi_n(\eta, \varphi_n), \quad (19)$$

the Schrödinger equation for each mode is

$$i \frac{d}{d\eta} \chi_n(a, \varphi_n) = -\frac{1}{2} \left[\partial_n^2 - \Omega_n^2 \varphi_n^2 \right] \chi_n(a, \varphi_n). \quad (20)$$

As usual, we now assume the gaussian ansatz for $\chi_n(\eta, \varphi_n)$:

$$\chi_n(\eta, \varphi_n) = A_n(\eta) \exp \left[i \alpha_n(\eta) - B_n(\eta) \varphi_n^2 \right], \quad (21)$$

where $A_n(\eta)$ and $\alpha_n(\eta)$ are real, while $B_n(\eta)$ may be complex, namely, $B_n(\eta) = B_{nR}(\eta) + i B_{ni}(\eta)$.

After integration of the scalar field modes, we can define the reduced density matrix ρ_{ras} :

$$\rho_r^{\alpha\beta}(a, a') = \prod_{n=1}^{\infty} \rho_{rn}^{\alpha\beta}(\eta, \eta') = \prod_{n=1}^{\infty} \int d\varphi_n \chi_n^\alpha(\eta, \varphi_n) \chi_n^\beta(\eta, \varphi_n). \quad (22)$$

where α and β symbolizes the two different classical geometries.

It is convenient to introduce the following change of variable in order to characterize the wave function of each mode:

$$B_m = -\frac{1}{2} \frac{\dot{g}_m}{g_m}. \quad (23)$$

where g_N is the wave function that represents the quantum state of the universe being also the solution of the differential equation

$$\ddot{g}_m + \Omega_m^2 g_m = 0, \quad (24)$$

Ω_m can be the complex energy Ω_n in our treatment.

In the more general case we use an arbitrary initial state $|0, 0\rangle$, instead of $|0, in\rangle$. From the discussion presented in the Introduction, and from Ref. [11] we know that, in a generic case, an infinite set of complex poles does exist. Then we must change (16) by $k^2 = k_n^2$ ($n = 0, 1, 2, \dots$), where these are the points where the infinite poles are located in the complex plane k^2 ; thus, Ω_n^2 now reads as

$$\Omega_n^2 = m^2 a^2 + k_n^2. \quad (25)$$

We will consider the asymptotic (or adiabatic) expansion of function g_N when $a \rightarrow +\infty$ in the basis of the out modes. g_N is the wave function that represents the state of the universe, corresponding to the arbitrary initial state; its expansion reads

$$g_m = \frac{P_m}{\sqrt{2\Omega_m}} \exp[-i \int_0^\eta \Omega_m d\eta] + \frac{Q_m}{\sqrt{2\Omega_m}} \exp[i \int_0^\eta \Omega_m d\eta], \quad (26)$$

where P_m and Q_m are arbitrary coefficients showing that $|0, 0\rangle$ is really arbitrary.

It is obvious that if all the Ω_m are real, like in the case of the Ω_k , (26) will have an oscillatory nature, as well as its derivative. This will also be the behaviour of B_k . Therefore the limit of B_k when $\eta \rightarrow +\infty$ will be not well defined even if B_k itself is bounded.

But if Ω_m is complex the second term of (26) will have a damping factor and the first a growing one. In fact, the complex extension of Eq. (26) (with $m = n$) reads

$$g_n = \frac{P_n}{\sqrt{2\Omega_n}} \exp[-i \int_0^\eta \Omega_n d\eta] + \frac{Q_n}{\sqrt{2\Omega_n}} \exp[i \int_0^\eta \Omega_n d\eta]. \quad (27)$$

Therefore when $\eta \rightarrow +\infty$ we have

$$B_n \approx -\frac{i}{2} \frac{\dot{g}_n}{g_n} = \frac{1}{2} \Omega_n. \quad (28)$$

Then we have two cases:

i) $\Omega_N = \Omega_k \in \mathcal{R}^+$ for the real factors. Then we see that when $\eta \rightarrow +\infty$, the r.h.s. of (22) is an oscillatory function with no limit in general. We only have a good limit for some particular initial conditions [10] (as $Q_m = 0$ or $P_m = 0$).

ii) $\Omega_m = \Omega_n = E_n - \frac{i}{2} \tau_n^{-1} \in \mathcal{C}$ for the complex factors. If we choose the lower Hardy class space Φ_- to define our rigged Hilbert space we will have a positive imaginary part, and

there will be a growing factor in the first term of (26) and a damping factor in the second one. In this case, for $a \rightarrow +\infty$, we have a definite limit:

$$B_n = \frac{1}{2}\Omega_n. \quad (29)$$

From equations (11), (17) and (29) we can compute the expression for B_n for both semiclassical solutions α and β :

$$B_n(\eta, \alpha) = B_n(\eta, \beta) = \frac{\sqrt{2}}{4} \left[m^2 B^2 \eta^2 + \left(m^4 B^4 \eta^4 + 4m^2 B^2 \left(n + \frac{1}{2} \right)^2 \right)^{\frac{1}{2}} \right]^{\frac{1}{2}} \quad (30)$$

$$-i \frac{\frac{\sqrt{2}}{2} m B \left(n + \frac{1}{2} \right)}{\left[m^2 B^2 \eta^2 + \left(m^4 B^4 \eta^4 + 4m^2 B^2 \left(n + \frac{1}{2} \right)^2 \right)^{\frac{1}{2}} \right]^{\frac{1}{2}}}.$$

Now we will see, making the exact calculations, that in the limit $\eta \rightarrow \infty$ there is necessarily decoherence for:

- a) different classical geometries ($\alpha \neq \beta$), i.e. $|\rho_r^{\alpha\beta}(\eta, \eta')| \rightarrow 0$ when $\eta \rightarrow \infty$.
- b) for the same classical geometry if the times η and η' are different, namely $|\rho_r^{\alpha\alpha}(\eta, \eta')| \rightarrow 0$ and $|\rho_r^{\beta\beta}(\eta, \eta')| \rightarrow 0$ when $\eta \rightarrow \infty$.

From equations (21) and (22) we obtain:

$$\rho_{rn}^{\alpha\beta}(\eta, \eta') = \left(\frac{4 B_{nR}(\eta, \alpha) B_{nR}(\eta', \beta)}{[B_n^*(\eta, \alpha) + B_n(\eta', \beta)]^2} \right)^{\frac{1}{4}} \exp[-i\alpha_n(\eta, \alpha) + i\alpha_n(\eta', \beta)]. \quad (31)$$

First, we will study decoherence for a) the same semiclassical solution but for different conformal times. Therefore we will calculate the asymptotic behavior ($\eta, \eta' \rightarrow \infty$) of $|\rho_{rn}^{\alpha\alpha}(\eta, \eta')|$, that reads:

$$|\rho_{rn}^{\alpha\alpha}(\eta, \eta')| \cong \left[\frac{4\eta\eta'}{[\eta + \eta']^2} \right]^{\frac{1}{4}}. \quad (32)$$

Making the following change of variable: $\frac{\eta - \eta'}{2} = \Delta$; $\frac{\eta + \eta'}{2} = \bar{\eta}$ with $\Delta \ll 1$ we obtain:

$$|\rho_{rn}^{\alpha\alpha}(\eta, \eta')| \cong \left[1 - \left(\frac{\Delta}{\bar{\eta}} \right)^2 \right]^{\frac{1}{4}}. \quad (33)$$

Since $|\rho_{rn}^{\alpha\alpha}(\eta, \eta')| \leq 1$ with the equality only if $\eta = \eta'$, it is easy to see from Eq. (22) that $|\rho_r^{\alpha\alpha}(\eta, \eta')|$ is equal to zero if $\eta \neq \eta'$. This means that the reduced density matrix

has diagonalized perfectly, i.e. we have achieved perfect decoherence. However, it is known [2,3,10] that perfect decoherence also implies that the Wigner function has an infinite spread, so we cannot say that the system is classical.

On the other hand, in Refs. [12,13] working with the consistent histories formalism made the assumption that exactly consistent sets of histories must be found very close to an approximately consistent set. In fact we have found the exact consistent set of histories, so it would be reasonable to say that there are many approximate consistent sets near of it. Although we are not working with this formalism, we can consider geometries that this statement is also valid in our case. Then, having an exact consistent set of histories means in our formalism exact decoherence. So, we can try to find the approximate decoherence (i.e. the approximate consistent sets) near the exact one.

IV. APPROXIMATE DECOHERENCE AND CLASSICAL CORRELATIONS

If we introduce a cutoff, N in Eq. (22) at some very large value of n , the reduced density matrix is not diagonal anymore, i.e. we obtain an approximate decoherence. Let us postpone for the next section the discussion about the value and nature of N . Thus we obtain if $\eta \approx \eta'$:

$$|\rho_r^{\alpha\alpha}(\eta, \eta')| = \left| \prod_{n=1}^N \rho_{rn}^{\alpha\alpha}(\eta, \eta') \right| \approx \exp - \left[\frac{N}{4} \left(\frac{\Delta}{\bar{\eta}} \right)^2 \right]. \quad (34)$$

From the last equation, we observe that the reduced density matrix turns out to be a gaussian of width σ_d where :

$$\sigma_d = \frac{2 \bar{\eta}}{N^{\frac{1}{2}}}. \quad (35)$$

Thus, it must be $\sqrt{N} \gg 1$ in order to obtain decoherence.

From equations (30) and (31) we compute $|\rho_r^{\beta\beta}(\eta, \eta')|$ and b) $|\rho_r^{\alpha\beta}(\eta, \eta')|$ and obtain for $\eta \rightarrow \infty$ as in eq. (32):

$$|\rho_{rn}^{\beta\beta}(\eta, \eta')| = |\rho_{rn}^{\alpha\beta}(\eta, \eta')| \cong \left[\frac{4 \eta \eta'}{[\eta + \eta']^2} \right]^{\frac{1}{4}}. \quad (36)$$

So, following the same steps we did for $|\rho_{rn}^{\alpha\alpha}(\eta, \eta')|$ [Eqs. (32) to (35)] we can see that the "decoherence conditions" (Eq. 35) are the same for a) case: different conformal times, and b): for different classical geometries. It is easy to see that we can follow the same steps for $|\rho_{rn}^{\alpha\beta}(\eta, \eta')|$ since from eq. (30) $B_n(\eta, \alpha) = B_n(\eta, \beta)$.

At this point we will analyze the existence of correlations between coordinates and momenta using Wigner function criterion [1]. Since correlations between coordinates and momenta should be examined "inside" each classical branch, we compute Wigner function associated with each semiclassical solution. The Wigner function associated with the reduced density matrix given by equations (22) and (31) is [10]:

$$F_W^{\alpha\alpha}(a, P) \cong C^2(\eta) \sqrt{\frac{\pi}{\sigma_c^2}} \exp \left[-\frac{\left(P - M^2 \dot{S} + \sum_{n=1}^N \left(\dot{\alpha}_n - \frac{\dot{B}_{ni}}{4B_{nR}} \right) \right)^2}{\sigma_c^2} \right], \quad (37)$$

where

$$\sigma_c^2 = \sum_{n=1}^N \frac{|\dot{B}_n|^2}{4B_{nR}^2}. \quad (38)$$

We can predict strong correlation when the centre of the peak of Wigner function is large compared to the spread, i.e., when:

$$\left(M^2 \dot{S} - \sum_{n=1}^N \left(\dot{\alpha}_n - \frac{\dot{B}_{ni}}{4B_{nR}} \right) \right)^2 \gg \sigma_c^2. \quad (39)$$

Using the same approximation we made for calculating the reduced density matrix, we obtain the following expression for the width of Wigner function:

$$\sigma_c^2(\eta, \alpha) \cong \frac{N}{4\eta^2}. \quad (40)$$

We can see that the σ_c is the inverse of σ_d (Eq. (34)), showing the antagonistic relation of decoherence and correlations [10].

We also calculate the centre of the peak of Wigner function, namely:

$$\left(M^2 \dot{S} - \sum_{n=1}^N \left(\dot{\alpha}_n - \frac{\dot{B}_{ni}}{4B_{nR}} \right) \right)^2 \cong m^2 B^2 N^2 \eta^2. \quad (41)$$

From equations (40) and (41) we it is posible to see the behavior of the centre of the peak and the width of Wigner's function in the limit $\eta \rightarrow \infty$. Thus the condition for the existence of correlations turns out to be:

$$N \gg \frac{1}{m^2 B^2 \eta^4}. \quad (42)$$

So, if the value of the cutoff is such that $N \gg 1$ and $N \gg \frac{1}{m^2 B^2 \eta^4}$ we can say that the sistem behaves classically: the off-diagonal terms of the reduced density matrix are exponentially smaller than the diagonal terms while we can predict strong correlations between $a(\eta)$ and its conjugate momenta.

A. Decoherence and Correlations with a specific value for the cutoff

In this subsection we propose and discuss a particular value for the cutoff N , using a relevant physical scale of the theory, namely, the Planck scale.

As we already have mentioned, it has been studied that stable and unstable particles are created in universe expansion [7,6,9]. But, in this work, we have used only the contribution of the unstable particles (the poles of the S matrix) to verify the emergence of the classical behavior. Thus, a reasonable choice for the value of N might be to consider in Eq. (34) only those unstable particles (poles) whose mean life is bigger than Planck's time ($t_p = M^{-1}$ in our units). This implies that particles with smaller life time will be considered to be outside the domain of our semiclassical quantum gravity model.

In order to calculate the mean life of each pole we have to transform equations (17), (18) and (30) to the non-rescaled case, namely the physical energy is $\frac{\Omega_n}{a}$ and the physical decaying time is $\tau'_n = a\tau_n$. Thus from (18) we obtain for $\eta \rightarrow \infty$ the mean life of the unstable state n:

$$\tau'_n = \frac{B \eta_{out}^2}{\left(n + \frac{1}{2}\right)}. \quad (43)$$

Thus, with this choice, we consider in Eq. (34) only those unstable particles with mean life:

$$\tau'_n = \frac{B \eta^2}{\left(n + \frac{1}{2}\right)} > \frac{1}{M} = t_p. \quad (44)$$

Therefore the value of the cutoff turns out to be $N = M B \eta^2$. It could be argued that this particular value of N depends of the conformal time η , but it should be noted that $\frac{N}{a^2(\eta)}$ does not depend on η anymore. Therefore, $N = N(\eta)$ should be regarded as a consequence of the universe expansion. The reduced density matrix (Eq. (34)) turns out to be a Gaussian of width σ_d where:

$$\sigma_d = \frac{2 \eta}{N^{\frac{1}{2}}} = \frac{2}{(M B)^{\frac{1}{2}}}; \quad (45)$$

and, as $\eta = \left(\frac{2t}{B}\right)^{\frac{1}{2}}$, we obtain the following expression for the ratio $\frac{\sigma}{\eta}$ as a function of t .

$$\frac{\sigma_d}{\eta} = \sqrt{\frac{2}{M t}} \approx \sqrt{\frac{t_p}{t}}. \quad (46)$$

Therefore the off-diagonal terms will be exponentially smaller than the diagonal terms for $t \gg \frac{1}{M} = t_p$.

With $N = M B \eta^2$, we obtain the following expression for eq. (39):

$$m^2 M B^3 \eta^6 \gg 1. \quad (47)$$

Writing the last equation as a function of the physical time t , we obtain the condition for the existence of strong correlations :

$$t \gg \left(\frac{t_p}{8 m^2}\right)^{\frac{1}{3}}. \quad (48)$$

V. CONCLUSIONS

We have shown that the S-matrix of a quantum field theory in curved space model has an infinite set of poles. The presence of these singularities produce the appearance of unstable ideal generalized states (with complex eigenvalues) in the Universe evolution. The corresponding eigenvectors are Gamow vectors and produce exponentially decaying terms.

The best feature of these decaying terms is that they simplify and clarify calculations. The Universe expansion leads to decoherence if this expansion produces particles creation as well. Our unstable states enlarge the set of initial conditions where we can prove that decoherence occurs. In fact, the damping factors allow that the interference elements of the reduced density matrix disappear for almost any non-equilibrium initial condition of the matter fields. Following the standard procedures, we have also shown that the unstable ideal generalized states satisfy the correlation conditions, which, with the decoherence phenomenon, are the origin of the semiclassical Einstein equations.

The conditions about decoherence and correlations were imposed by means of an ultraviolet cutoff, N , related with the energy scale where the semiclassical approximation is taken as valid. The introduction of this cutoff is relevant in order to preserve both necessary conditions for classicality: decoherence plus correlations. Without the presence of the cutoff the infinite set of unstable codes destroy the classical correlation and the semiclassical limit would be untenable.

Decoherence is the key to understanding the relationship between the arrows of time in cosmology. In the context of quantum open systems, where the metric is viewed as the “system” and the quantum fields as the “environment,” decoherence is produced by the continuous interaction between system and environment. The non-symmetric transfer of information from system to environment is the origin of an entropy increase (in the sense of von Neumann), because there is loss of information in the system, and of the time asymmetry in cosmology, because growth of entropy, particle creation and isotropization show a tendency towards equilibrium. However, decoherence is also a necessary condition for the quantum to classical transition. In the density matrix formulation, decoherence appears as the destruction of interference terms and, in our model, as the transition from a pure to a mixed state in the time evolution of the density matrix associated with the RW metric; the interaction with the quantum modes of the scalar fields is the origin of such a non-unitary evolution.

It is interesting to note that, in the cosmological model we considered, unstable particle

creation and decoherence are the effect of resonances between the evolutions of the scale factor a and the free massive field, which is, on the other hand, the origin of the chaotic behaviour in the classical evolution of the cosmological model [14]. This observation opens a new and interesting path in the study of the relationship between classical chaotic models and the decoherence phenomena.

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